Since when $x = y = 0$, $p = \dfrac{b}{a}$, therefore $c = \dfrac{b + \sqrt{a^2 + b^2}}{a^{(n+1)/n}}$.

Hence, also

$$y_a = \frac{cn}{2(n+1)} a^{(n+1)/n} - \frac{n}{2c(n-1)} a^{(n-1)/n}.$$

Therefore, finally, we have

$$y = \frac{cn}{2(n+1)} (a^{(n+1)/n} - (a-x)^{(n+1)/n}) - \frac{n}{2c(n-1)} (a^{(n-1)/n} - (a-x)^{(n-1)/n}).$$

We see that the curve K_1 cuts BC at the point (a, y_a) and, since for $n = 1$ $y_a = \infty$, it follows that n must be greater than 1 if the hound can catch the fox.

If we assume the side of the square AB equal to 2, then we have $a = 1$, $b = 0$ and therefore $c = \pm 1$, where we have to take $c = -1$, because y' is always positive and we get

$$y = \frac{n}{2(n-1)} (1 - (1-x)^{(n-1)/n}) - \frac{n}{2(n+1)} (1 - (1-x)^{(n+1)/n}),$$

and

$$y_a = \frac{n}{2(n-1)} - \frac{n}{2(n+1)}.$$

As long as $y_a < 2$, the hound catches the fox on the side BC of the square, and we find the smallest n from $2 = \dfrac{n}{2(n-1)} - \dfrac{n}{2(n+1)}$, that is, $n = 1.3680$. If $n < 1.3680$, then we find the point $O_1 \equiv (x_1, y_1)$ whose tangent passes through C and lay a new system of axes X_1Y_1 for the curve K_2. Here is $a = 2 - y_1$, $b = 1 - x_1$, so that we find the equation of the curve K_2 and can find y_a in which alone we are interested. If the hound does not catch the fox on the side CD we have to repeat the process.

We find O_1 by use of the equation $2 = y + y'(a - x)$.

By substituting the above values for y and y', we find

$$\frac{2n}{n^2 - 1} - 4 = \frac{(1-x)^{(n+1)/n}}{n+1} + \frac{(1-x)^{(n-1)/n}}{n-1}.$$

If we assume $(1 - x)^{1/n} = U$, we have the form $a = bU^m + cU^{m'}$ from which we find U, and hence x and O_1.

Also solved by Barnem Libby.

334. Proposed by Elmer Schuyler, Brooklyn, N. Y.

Solve

$$\frac{\partial^2 T}{\partial u \partial v} + \frac{2v}{u^2 + v^2 + 1} \frac{\partial T}{\partial u} + \frac{2u}{u^2 + v^2 + 1} \frac{\partial T}{\partial v} = 0.$$

SOLUTION BY R. D. CARMICHAEL, Indiana University.

The equation may be written in the form

$$\frac{\partial}{\partial v}\left(\frac{\partial T}{\partial u}+\frac{2u}{u^2+v^2+1}T\right)+\frac{2v}{u^2+v^2+1}\left(\frac{\partial T}{\partial v}+\frac{2u}{u^2+v^2+1}T\right)=0.$$

This equation may be viewed as an ordinary differential equation for determining the quantity in parenthesis; the arbitrary "constant" is then a function of u. Solving this equation, we have

$$\frac{\partial T}{\partial u}+\frac{2u}{u^2+v^2+1}T=\frac{\bar{\psi}(u)}{u^2+v^2+1};$$

or

$$(u^2+v^2+1)\frac{\partial T}{\partial u}+2uT=\bar{\psi}(u).$$

Hence

$$(u^2+v^2+1)T=\int\bar{\psi}(u)du+\varphi(v)=\psi(u)+\varphi(v).$$

Therefore, we have

$$T=\frac{\psi(u)+\varphi(v)}{u^2+v^2+1},$$

where $\psi(u)$ and $\varphi(v)$ are arbitrary functions of u and v respectively.

Solved in a different manner by the Proposer.

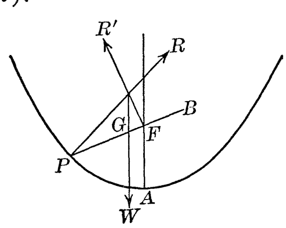
MECHANICS.

267. Proposed by G. H. LIGHT, Purdue University.

A parabolic curve is placed in a vertical plane with its axis vertical and vertex downwards, and inside it, and against a peg in the focus, and against the concave arc, a smooth uniform and heavy beam rests; required the position of equilibrium. [From Bowser's Mechanics, Ex. 37, p. 96.]

SOLUTION BY CHRISTIAN HORNUNG, Heidelberg University, Tiffin, O.

Let PB be the beam of length l, of weight W, resting on the peg at the focus, F; let $AF = p$, $AFP = \theta$, and $GP = l/2$. From the polar equation of the parabola we have $FP = 2p/(1 + \cos\theta)$.



Since all the surfaces are smooth the resistances, R and R', at P and F respectively, are normal and therefore the angle $BPR = \theta/2$.

Using the equations $\Sigma x = 0$, $\Sigma y = 0$ and Σ(moments about F) $= 0$, which